



**Thermal-wind dispatch programming using stochastic optimal power  
flow with discrete variables**

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#### ABSTRACT

In this study, results for a discrete and stochastic optimal power flow (DSOPF) model inserting wind generators for electricity generation are presented. The objective function of the DSOPF model aims to minimize the cost of active power generation in the system. This cost is composed of nonlinear and non-differentiable functions representing thermoelectric generators, due to the valve loading point effect (VLPE) and by integral functions representing wind generators. The DSOPF model is solved using a modified logarithmic barrier (MLB) primal-dual interior/exterior point method with a predictor-corrector procedure and inertia correction. This method is deterministic and, unlike metaheuristic methods, guarantees local optimal solutions and calculates the price of the energy generated from the dual variables of the problem. To get the results, a 30-bus IEEE system was used to simulate the grid in a city in northeastern Brazil, which experiences significant variation in average wind speed throughout the four seasons of the year. The results show a reduction in generation costs through the substitution of thermal generators by wind generators. The wind power, which is uncertain due to wind speed, is represented and integrated using the Weibull probability distribution function. The article contributes to the Sustainable Development Goals, particularly Goal 7, by minimizing the total cost of energy generation from renewable sources related to wind energy in electric power systems.

**KEY WORDS:** Stochastic Optimal Power Flow. Clean energy. Primal-dual interior/exterior point method.

#### 1 INTRODUCTION

The use of renewable energy sources has been gaining global prominence and major incentives to make electrical grids cleaner. In Brazil, wind energy already represents about 12% of the Brazilian electricity matrix and we are already in 6th place in the Global Capacity Installed Ranking, according to data from Associação Brasileira de Energia Eólica (ABEEÓLICA) (2022). The insertion of wind power requires studies, as it can cause uncertainties in meeting demand in the sector.

A wind power schedule is needed that better represents reality and that meets the demand, considering that wind power generation for a generator depends on the wind speed. So, although it is a clean source, it has uncertainties that need to be addressed.

The same does not happen with thermal power plants powered by natural gas, coal, or fossil fuels. To meet a certain demand, they only need the raw material. However, these forms of electricity generation emit large amounts of  $CO_2$ , which pollute the environment, and are not a clean source of energy. In Brazil, according to data from Instituto Estadual de Meio Ambiente (IEMA) (2022), there was an increase in generation from thermal power plants, from 15% in 2020 to 20% in 2021.

One way to address the uncertainty of wind speed and contribute to the security and meeting of demand in the electricity sector with reduced emissions is using the Weibull probability distribution function (WPDF) to calculate the probability of occurrence of wind power generation. This function has two parameters, the shape and scale, which in this case are determined from the average speeds and standard deviation of a certain period of the year in a certain region. This wind speed probability distribution curve allows the calculation of wind power from the variation of wind speed in the considered period and determines a probability distribution curve for the scheduled wind power. In this way, the wind power scheduling of a Discrete and Stochastic Optimal Power Flow (DSOPF) problem in an electric power system is carried out in a manner consistent with the region in which the system is installed.

In this study, a simulation of a 30-bus IEEE system, installed in the city of São Gonçalo do Amarante (CE), was carried out and the results presented. The wind power scheduling follows the average speed and the probability distribution curve for each season of the year.

The method used to solve the DSOPF problem and obtain the results was the modified logarithmic barrier (MLB) interior/exterior point predictor-corrector primal-dual, which considers the inertia correction procedure to address the multimodality of the problem and a strategy of auxiliary variables, to deal with the non-differentiability of the thermal cost function. This method is deterministic and, in addition to minimizing the total cost of the thermo-wind generation system, has the advantage of calculating the dual variables associated with the marginal cost or energy price at each generator, unlike other methods used to solve this problem, such as meta-heuristics, which do not give this possibility.

## 2 OBJECTIVE

In this study, the objective of the DSOPF problem is to minimize the costs of thermal and wind generation. In this problem, the function that represents the thermal costs is nonlinear, non-convex, and non-differentiable due to the insertion of valve loading point effect in it, which is represented by sinusoidal absolute value functions.

To represent the wind generation cost function, the linear, reserve and penalty cost functions are considered, in which the latter two are represented by integral functions defined from the underestimation and overestimation of wind power generation, expressed through the WPDF.

The objective is to solve the DSOPF problem and present results that can be used to guarantee demand satisfaction at the lowest cost, determining the prices of thermal or wind energy generation for each bus in the power system. In this sense, the main objectives of the study are:

- Solve a stochastic optimal power flow model with discrete variables through a deterministic method.
- Minimize the costs of thermal and wind generation and compare the scheduling of the generated power for thermal and wind generators according to the average wind speed.
- Determine the prices of thermal and wind generation from the dual variables of the deterministic MLB method used.

## 3 WIND POWER AND THE WPDF

In the DSOPF model used in this work and presented in the next section, one of the variables is wind power. Wind power is calculated from wind speed, which, as it is not controllable and alleatory, makes the model stochastic.

One way to determine the probability of occurrence of wind power in a given region is through the WPDF. This function is used to calculate the costs of wind generation.

The following is the Weibull probability density function (Wpdf) for determining the probability of occurrence of a wind speed, which is a continuous random variable and thus has a density function. As mentioned above, the random variable present in the model is wind power, so the calculation of wind power from wind speed is presented. Next, the Wpdf is presented, which is a transformation of the Wpdf in terms of wind power, used for the costs of wind generation.

### 3.1 Weibull probability density function (Wpdf)

The Wpdf estimates the probability of occurrence of wind speed in a given period or region from two parameters calculated from the mean speed and standard deviation of a database. Following is the Wpdf, Equation 3.1, Hetzer, David e Bhattarai (2008):

$$f^v(v) = \left(\frac{k}{c}\right) \left(\frac{v}{c}\right)^{(k-1)} e^{-\left(\frac{v}{c}\right)^k}, 0 < v < \infty \quad (3.1)$$

in which  $v$  is the wind speed,  $c$  is the scale factor in a given location, related to the mean wind speed and  $k$  is the shape factor in a given location, referring to the uniformity of the distribution of wind speed values.

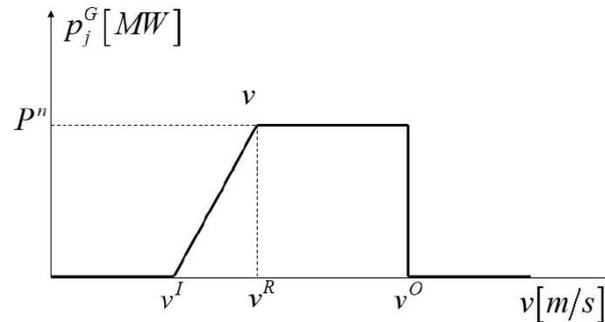
### 3.2 Wind power calculation

For the DSOPF model, the random variable is the wind power. The calculation is based on the physical constraints of a wind turbine, such that the wind power as a function of wind speed is defined in Equation (3.2) and represented in Figure 1:

$$P_j^G = \begin{cases} 0 & \text{para } v < v^I \text{ e } v > v^O \\ P^n \frac{(v - v^I)}{(v^R - v^I)} & \text{para } v^I \leq v \leq v^R \\ P^n & \text{para } v^R \leq v \leq v^O \end{cases} \quad (3.2)$$

in which  $P^n$  is the nominal power of the generator,  $v^I$  is the minimum wind speed for the start of operation of the wind turbine,  $v^R$  is the wind speed at which the wind turbine reaches the nominal power  $P^n$  and  $v^O$  is the cut-in wind speed, which ceases the operation of the wind turbine.

Figure 1 – Output of active wind power as a function of wind speed.



Source: (SOUZA, 2020)

### 3.3 The WPDF according to the wind power

In this study, as in the study by Souza (2020), a transformation of the Wpdf according to wind speed to a WPDF for the calculation of the probabilities of occurrence of wind power was used, as used in the study by Mishra, Singh e Rokadia (2015) and presented in Equation (3.3):

$$f^w(p_j^G) = \frac{k(v^R - v^I)}{cP^n} \left( \frac{p_j^G(v^R - v^I) + P^n v^I}{cP^n} \right)^{(k-1)} e^{-\left( \frac{p_j^G(v^R - v^I) + P^n v^I}{cP^n} \right)^k}, 0 < p_j^G < P^n \quad (3.3)$$

From the WPDF as a function of wind power, the highest probability of occurrence of the wind power for a given period or region can be determined using the shape  $k$  and scale  $c$  parameters. This function is then used in the wind generation cost calculation.

### 3.4 Wind generation costs

The costs associated with wind generation are: the linear cost ( $C^L$ ), the penalty cost ( $C^P$ ) and the reserve cost ( $C^R$ ), where  $\Omega_E$  is the set of wind generators and  $\omega^R$  and  $\omega^P$  are the weighting factors, as proposed by Souza (2020). The wind generation cost is expressed by Equation (3.4):

$$C_j^E(p_j^G) = C_j^L(p_j^G) + \omega_j^R C_j^R(p_j^G) + \omega_j^P C_j^P(p_j^G), \forall j \in \Omega_E \quad (3.4)$$

#### 3.4.1 Linear Cost

This cost is directly associated with the scheduled wind power, given by Equation (3.5):

$$C_j^L(p_j^G) = d_j p_j^G \quad (3.5)$$

where  $d_j$  is the linear cost coefficient of wind turbines.

### 3.4.2 Reserve Cost

The reserve cost is associated with the overestimation of wind generation. Based on the WPDF, the power scheduled by the MLB method can be checked to see if it is above the highest probability of occurrence. If this occurs, the reserve cost increases. The reserve cost is presented by Equation (3.6):

$$C_j^R(p_j^G) = K_j^R \int_0^{p_j^G} (p_j^G - w) f^W(w) dw \quad (3.6)$$

where  $K^R$  is the reserve coefficient and  $f^W$  is the function described by Equation (3.3).

### 3.4.3 Penalty Cost

The penalty cost is associated with the underestimation of wind generation. That is, if the wind power scheduling is below the highest probability of occurrence according to the WPDF for the period or region, the penalty cost increases, as wind power is being left ungenerated. The penalty cost is presented by Equation (3.7):

$$C_j^P(p_j^G) = K_j^P \int_{p_j^G}^{p_j^N} (w - p_j^G) f^W(w) dw \quad (3.7)$$

where  $K^P$  is the penalty coefficient and  $f^W$  is the function described by Equation (3.3).

## 4 THE DISCRETE AND STOCHASTIC OPTIMAL POWER FLOW PROBLEM

The DSOPF problem considered here is presented by Schimidt (2022). The objective function of the problem, represented by Equation (4.1.a), aims to minimize the costs of thermal and wind power generation. The continuous variables of the problem are: the active powers ( $p^G$ ) of the generating buses, the magnitudes ( $V$ ) and angles ( $\theta$ ) of the voltage at the buses; and the discrete variables are: power transformer tap ratios ( $t$ ) and the capacitor and shunt reactor bank ( $b^{sh}$ ). The DSOPF model used is presented below.

$$\text{Min} \sum_{i \in \Omega_T} C_i^T(p_i^G) + \sum_{j \in \Omega_E} C_j^E(p_j^G) \quad (4.1.a)$$

$$s.t. \quad \sum_{m \in \Omega_k} P_{km}(V, \theta, t) - p_k^G + P_k^C = 0, \forall k \in \Omega_T \cup \Omega_E \quad (4.1.b)$$

$$\sum_{m \in \Omega_k} P_{km}(V, \theta, t) - P_k^G + P_k^C = 0, \forall k \in \Omega_C \quad (4.1.c)$$

$$\sum_{m \in \Omega_k} Q_{km}(V, \theta, t) - Q_k^G + Q_k^C - Q_k^{sh}(V, b_k^{sh}) = 0, \forall k \in \Omega_C \quad (4.1.d)$$

$$Q_k^{G_{\min}} \leq q_k^G(V, \theta, t, b_k^{sh}) \leq Q_k^{G_{\max}}, \forall k \in \Omega_T \cup \Omega_E \quad (4.1.e)$$

$$P_k^{G_{\min}} \leq p_k^G \leq P_k^{G_{\max}}, \forall k \in \Omega_T \cup \Omega_E \quad (4.1.f)$$

$$V_k^{\min} \leq V_k \leq V_k^{\max}, \forall k \in \Omega_B \quad (4.1.g)$$

$$t_{km} \in \Omega_{Dtap}, \forall k \in \Omega_{tap} \quad (4.1.h)$$

$$b_k^{sh} \in \Omega_{Dbsh}, \forall k \in \Omega_{bsh} \quad (4.1.i)$$

In the objective function, Equation (4.1.a), the first term is the cost function of thermal generators, expressed by Equation (4.2):

$$C_i^T(p_i^G) = a_i(p_i^G)^2 + b_i p_i^G + c_i + e_i \left| \text{sen}(f_i(P_i^{G_{\min}} - p_i^G)) \right| \quad (4.2)$$

This function considers the valve loading point effect, which makes the function non-convex and non-differentiable at these points.

The second term of the objective function, Equation (4.1.a), is the cost associated with wind generation, described by Equation (4.3):

$$C_j^E(p_j^G) = C_j^L(p_j^G) + \omega_j^R C_j^R(p_j^G) + \omega_j^P C_j^P(p_j^G) \quad (4.3)$$

in which

$$C_j^L(p_j^G) = d_j p_j^G \quad (4.4)$$

$$C_j^R(p_j^G) = K_j^R \int_0^{p_j^G} (p_j^G - w) f^W(w) dw \quad (4.5)$$

$$C_j^P(p_j^G) = K_j^P \int_{p_j^G}^{p_j^n} (w - p_j^G) f^W(w) dw \quad (4.6)$$

In the studies by Souza (2020) and Souza *et al.* (2022), in contrast to traditional approaches, weighting factors  $\omega_j^R$  and  $\omega_j^P$  are introduced in Equation (4.3) associated with reserve and penalty costs, respectively. These factors are used to weight the reserve and penalty costs in the objective function, Equation (4.1.a). The reserve and penalty parameters,  $K_j^R$  and  $K_j^P$ , respectively, present in Equations (4.5) and (4.6), represent the adopted reserve and penalty prices. In the model presented here, a distinction is made between the real prices ( $K_j^R$

and  $K_j^P$ ) and the weighting factors ( $\omega_j^R$  and  $\omega_j^P$ ), which are considered in Equations (4.3), (4.5) and (4.6).

In this way, the wind power scheduling is adjusted by the system operator based on the weighting factors ( $\omega_j^R$  and  $\omega_j^P$ ) without influencing the wind generation prices, which are adjusted by the reserve and penalty parameters ( $K_j^R$  and  $K_j^P$ ), respectively.

Constraint (4.1.b) corresponds to the power balance of the system generation buses, in which the generated active power  $p_k^G$  is a variable of the problem, in the case of wind generators, it is a random variable, which makes the model stochastic. In Equation (4.1.c) we have the balance of active power of the load buses, in which the active power  $P_k^G$  is a constant, for this reason the different nomenclature.

Constraint (4.1.d) represents the reactive power balance of the load buses of the problem and Equation (4.1.e) represents a constrained inequality with limits of reactive power generation for the generation buses of the problem.

Constraints (4.1.f) and (4.1.g) are inequality constraints of the continuous variables of the problem with their minimum and maximum limits of active power and operation of the magnitude of voltages at the buses, respectively.

Constraints (4.1.h) and (4.1.i) represent, respectively, the taps of the in-phase transformers and capacitor banks and shunt reactors of the system that must belong to their respective discrete sets.

The complete nomenclature containing the definition of indices, parameters, sets and variables can be found in Schimidt (2022).

## 5 METHODOLOGY

To solve the DSOPF problem presented in the previous section, strategies were developed for applying the MLB method, developed by Souza *et al.* (2022) and by Schimidt (2022):

- Strategy for treating the sinusoidal absolute value function related to the thermal generation cost function, as proposed by Bertsekas (1997), in which the absolute value in the objective function is replaced by an auxiliary variable to be minimized, which establishes an upper and lower limit in a bounded constraint of the function, disregarding modular terms;
- Strategy for determining the first and second order derivatives of the integrals that define the reserve and penalty costs considered for the wind power, through the Fundamental Theorem of Calculus, as proposed by Souza (2020) and Souza *et al.* (2022);
- Strategy for treating discrete variables using a sinusoidal penalty function as described by Soler, Asada e Costa (2013);
- Inertia correction strategy by Silva (2014) to treat the multimodality of the problem and determine only discrete minimum points.

The consideration of these strategies in the MLB method allows the application of this method to the solution of DSOPF problems defined considering Equations (4.1.a – 4.1.i).

The following results are presented for a 30-bus IEEE system that validate the application of the proposed model and the MLB method to the solution of the system.

## 6 RESULTS

For the simulation, the MLB method was implemented in Matlab, using Matlab R2016a, on a computer with an Intel Core i3 processor, with 4GB of RAM and Windows 10 Pro operating system. The main data of the IEEE 30-bus system used are available at <https://matpower.org/download/all-releases/>, while the data for the generating units are available in Pinheiro, Balbo e Nepomuceno (2019).

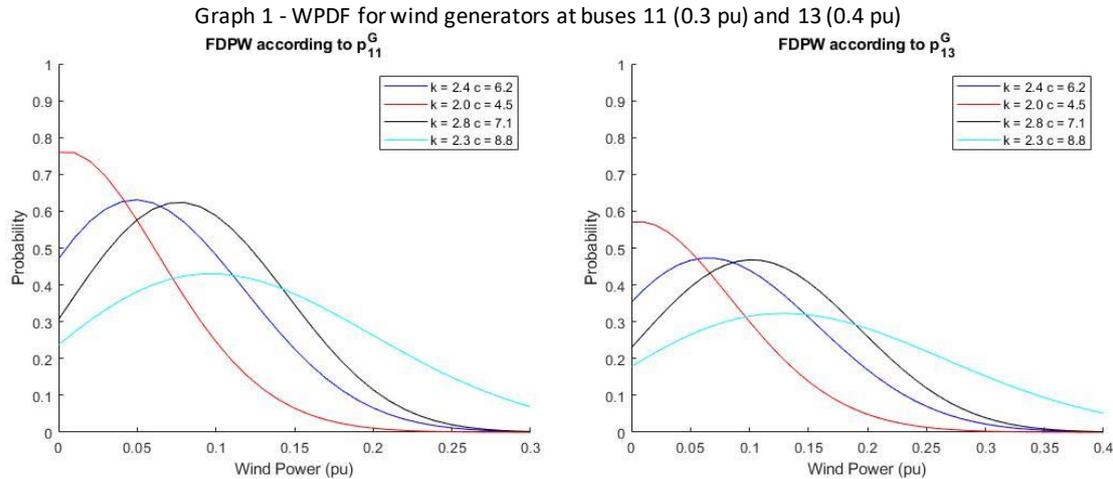
A simulation of a 30-bus IEEE system installed in the city of São Gonçalo do Amarante (CE) was carried out. In the first case, the results only came from the use of thermal generators for electricity generation. In the second case, the generator at Bus 13 was replaced by a wind turbine with the same nominal power (40 MW) and in the third case, another thermal generator, the generator at Bus 11, was replaced by a wind turbine with the same nominal power (30 MW).

In Cases 2 and 3, the parameters associated with WPDF are based on real values for the region of the city of São Gonçalo do Amarante (CE, Brazil), which are available on the website [http://www.cresesb.cepel.br/index.php?section=atlas\\_eolico&](http://www.cresesb.cepel.br/index.php?section=atlas_eolico&) (Table 1). According to the report released by Instituto Estadual de Meio Ambiente (IEMA) (2022), the city is in 6th place in the ranking of generation by thermal power plant, with the Porto do Pecém I Thermal Power Plant. The city was chosen because it presents a large variation in the average wind speed in different seasons of the year.

Table 1 - Data for the WPDF of energy production in the city of São Gonçalo do Amarante (CE, Brazil).

Seasons of the year	Dec – Feb (a)	Mar – May (b)	Jun – Aug (c)	Sep – Nov (d)
Shape parameter ( $k$ )	2.4	2.0	2.8	2.3
Scale parameter ( $c$ )	6.2	4.5	7.1	8.8
Average wind speed (m/s)	5.5	3.95	6.32	7.79

Source: Centro de Referência para Energia Solar e Eólica Sérgio Brito (CRESESB) (2013).



Tables 2 and 4 summarize the results and show: the generation dispatch; the system demand; the thermal costs, given by the quadratic costs and the cost of the valve loading point effect (VLPE); the costs of wind generation, given by the linear, reserve and penalty costs; the total costs, involving thermal and wind costs; the number of iterations and the computational times.

In Case 1, only with thermal generators, a thermal cost was obtained, equivalent to the total cost of 841.79 \$/h, meeting the demand of 293 MW. The MLB method determined a power of 12 MW for the thermal generator at Bus 13, which in Case 2 was replaced by a wind generator with a nominal power of 40 MW.

Table 2 shows that the power determined for the wind generator at Bus 13 varied according to the average speed for each period studied (2a – 2d); in the period with the lowest average wind speed (2b) of 3.95 m/s the power was 9.19 MW, generating a wind cost of 14.41 \$/h, which added to the thermal cost of 812.74 \$/h, made a total cost of 827.15 \$/h, which was below the total cost for Case 1 with only thermal generators.

In the period with the highest average wind speed (2d) of 7.79 m/s, the power determined for the wind generator was 20.36 MW, generating a wind cost of 33.19 \$/h, which, added to the thermal cost of 773.58 \$/h, made a total cost of 806.76 \$/h.

In all periods for Case 2, the total system cost was lower than the total cost when using thermal generators only, with the wind powers determined for the wind generator at Bus 13, as shown in Graph 1, close to the highest probability of wind occurrence in that region. Despite the uncertainty related to wind speed, wind powers are consistent with the time of year and region in which the system is installed can be determined.

Table 2 – Results of the IEEE 30-bus system – Cases 1 and 2

Case	1	2a	2b	2c	2d
$p_1^G$ (MW)	200.00	200.00	200.00	200.00	200.00
$p_2^G$ (MW)	36.92	42.45	45.68	40.59	35.89
$p_5^G$ (MW)	18.50	18.52	18.99	18.22	17.39
$p_8^G$ (MW)	14.52	10.00	10.00	10.00	10.00
$p_{11}^G$ (MW)	11.66	10.00	10.00	10.00	10.00
$p_{13}^G$ (MW) (Wind)	12.00	12.82	9.19	14.93	20.36
Demand (MW)	293.60	293.78	293.86	293.73	293.64
Quadratic Cost (\$/h)	805.40	761.60	773.83	754.67	737.47
VLPE Cost (\$/h)	36.38	38.29	38.2	37.80	36.11
<b>Thermal Cost (\$/h)</b>	<b>841.79</b>	<b>799.89</b>	<b>812.74</b>	<b>792.47</b>	<b>773.58</b>
Linear Cost (\$/h)	-	12.82	9.19	14.93	20.36
Reserve Cost (\$/h)	-	5.33	3.35	6.16	8.34
Penalty Cost (\$/h)	-	3.20	1.87	3.5	4.48
<b>Wind Power Cost (\$/h)</b>	<b>-</b>	<b>21.34</b>	<b>14.41</b>	<b>24.59</b>	<b>33.19</b>
<b>Total Cost (\$/h)</b>	<b>841.79</b>	<b>821.23</b>	<b>827.15</b>	<b>817.06</b>	<b>806.76</b>
Iterations	55	26	41	25	26
Time (s)	21.11	14.25	16.70	11.27	12.22

Source: Author, 2023.

Case 3 shows that the system operator can manipulate parameters to have greater control over the power and price of wind generation in the system, respecting the probability distribution curve for wind power in that region for each given period.

The MLB method, used to solve the DSOPF problem, has the advantage of calculating the primal and dual minimum solutions, in which the dual solutions are related to the energy prices (incremental costs) per bus, which are important for optimizing the generation scheduling. Tables 3 and 5 present information on the dual values obtained by the MLB method for the 30-bus IEEE system, showing that the incremental costs (derivative of the cost function) and nodal prices (Lagrange multipliers associated with the power balance equations at each bus) tend to have approximately equal values at each bus, following the principle of merit order dispatch.

This principle consists of dispatching the cheapest plants first, in order to minimize the operating costs of the electrical system. Exceptions occur when a unit reaches its upper or lower limit, so that the incremental costs and nodal prices tend to become different in this case.

The incremental costs and nodal prices, given in Table 3 for bus 13 in Case 2, follow the principle of merit order dispatch and show that the price of wind generation is the lowest in Case 2 for the period with the highest average wind speed (2d), with \$3.59/MW, compared to the other periods, and the highest price is for the period with the lowest average wind speed (2b), with \$3.81/MW.

Table 3 – Incremental costs and nodal prices for each bus for cases 1 and 2

Case	1		2a		2b		2c		2d	
Bus	Inc. (\$/MW)	Nod. (\$/MW)								
1	3.5000	3.4495	3.5000	3.4841	3.5000	3.5369	3.5000	3.4514	3.5000	3.3587
2	3.0339	3.6004	3.2358	3.6454	3.3488	3.6895	3.1707	3.6019	3.0061	3.5066
5	3.3046	3.8283	3.3143	3.8688	3.3740	3.9269	3.2776	3.8329	3.1736	3.7311
8	3.4938	3.7652	3.4168	3.8135	3.4168	3.8741	3.4168	3.7761	3.4168	3.6708
11	3.5931	3.7542	3.5000	3.7983	3.5000	3.8577	3.5000	3.7618	3.5000	3.6584
13	3.6000	3.7047	3.7453	3.7453	3.8093	3.8093	3.7061	3.7061	3.5964	3.5964

Source: Author, 2023.

For Case 3, keeping the 30-bus IEEE system, one more thermal generator was replaced by a wind generator, so the system now has 2 wind generators, present at Buses 11 and 13, with nominal powers of 30 MW and 40 MW, respectively.

As mentioned earlier, the system operator has greater control over the power and price of wind generation. This is due to the reserve and penalty parameters, which compose the reserve and penalty costs, respectively. These costs are added to the linear cost to constitute the wind cost.

To illustrate, in this Case 3, for Bus 11, the reserve parameter will be  $K_{11}^R = 2$  and the penalty parameter will be  $K_{11}^P = 1$ . In this way, the generation at Bus 11 has more weight for the security of the system, so proportionally, less wind power will be scheduled compared to the generation at Bus 13. And for Bus 13, the reserve parameter will be maintained at  $K_{13}^R = 1.5$  and the penalty parameter will be  $K_{13}^P = 2$ . In this case, the penalty parameter is higher, which increases the generation, because not generating wind power has a higher cost.

Table 4 shows, in Case 2, for all the studied periods (3a – 3d), the total system costs were lower than those in Case 1 (841.79 \$/h), in which only thermal generators were considered. In Case 3, because two wind generators were used, the total costs were lower for all periods compared to Case 2.

For the period with the lowest average wind speed (2b and 3b), the total system cost for Case 2b was 827.15 \$/h, while in Case 3b, 822.22 \$/h. In the period with the highest average wind speed (2d and 3d), the largest difference in costs was obtained, being in case 2d, 806.76 \$/h, and in Case 3d, 790.49 \$/h.

Table 4 clearly shows that, proportionally, the wind generator at Bus 13 had a larger wind power schedule than the wind generator at Bus 11, because as mentioned above, the penalty parameter, referring to the penalty cost of Bus 13, was higher compared to its reservation parameter, referring to the reserve cost.

In this way, the method was induced to schedule more wind power for the wind generator at bus 13 to minimize costs, as generating less wind power than the highest probability of occurrence, according to the WPDF in the period, made the penalty cost more expensive, increasing the wind power cost for bus 13. Thus, scheduling less wind power than the

highest probability of occurrence generates a penalty in the wind objective function that increases the total generation cost of the system.

The opposite occurred with the programming of wind power at bus 11, as the reservation parameter was higher than the penalty parameter. Therefore, generating more wind power than the highest probability of occurrence, according to the WPDF, made the reserve cost more expensive, as it is assumed that, if more wind power is scheduled than the period can provide, the reserve cost increases, and if that scheduled power does not occur, it will be necessary to activate reserve power to meet demand. The results of the power scheduled for Buses 11 and 13 are illustrated in Graph 1.

Table 4 – Results of the IEEE 30-bus system – Cases 1 and 3

Case	1	3a	3b	3c	3d
$p_1^G$ (MW)	200.00	200.00	200.00	200.00	200.00
$p_2^G$ (MW)	36.92	45.67	50.37	42.52	35.57
$p_5^G$ (MW)	18.50	19.00	19.68	18.53	17.29
$p_8^G$ (MW)	14.52	10.00	10.39	10.00	10.00
$p_{11}^G$ (MW) (Wind)	11.66	6.38	4.36	7.81	10.43
$p_{13}^G$ (MW) (Wind)	12.00	12.86	9.26	14.95	20.35
Demand (MW)	293.60	293.91	294.06	293.81	293.63
Quadratic Cost (\$/h)	805.40	741.32	761.10	729.36	703.68
VLPE Cost (\$/h)	36.38	38.92	39.51	38.31	35.94
<b>Thermal Cost (\$/h)</b>	841.79	780.24	800.61	767.68	739.63
Linear Cost (\$/h)	-	19.24	13.62	22.77	30.78
Reserve Cost (\$/h)	-	7.66	4.80	8.95	11.97
Penalty Cost (\$/h)	-	5.56	3.20	6.20	8.12
<b>Wind Power Cost (\$/h)</b>	-	32.46	21.62	37.92	50.86
<b>Total Cost (\$/h)</b>	841.79	812.71	822.22	805.61	790.49
Iterations	55	32	68	25	29
Time (s)	21.11	14.19	19.92	12.16	13.45

Source: Author, 2023.

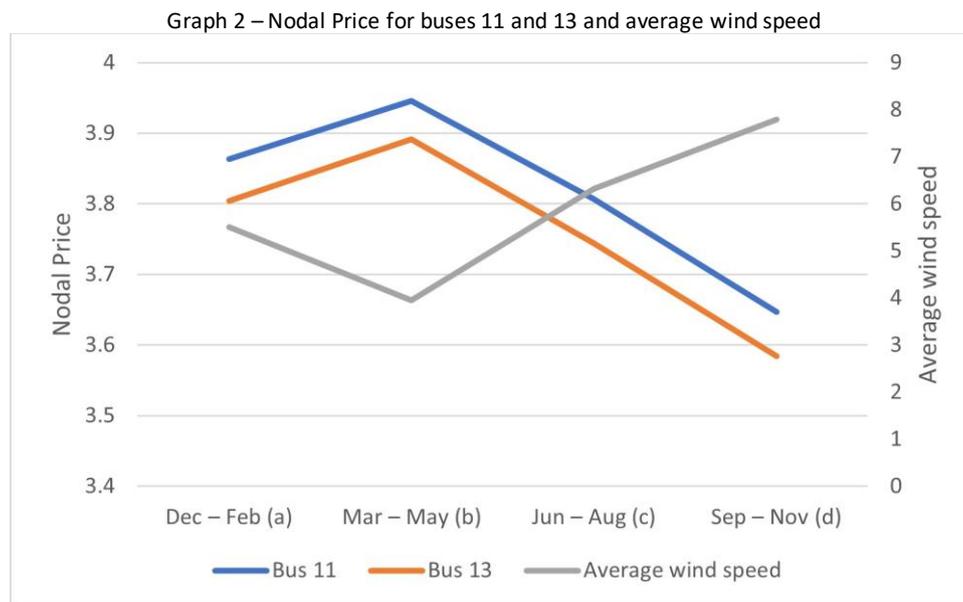
In Table 5, the incremental costs and nodal prices for Case 3 are presented. Because the wind generator at Bus 13 has a scheduled wind power proportionally greater than the scheduled wind power at Bus 11, the nodal prices for Bus 11 are higher in all periods compared to the nodal prices for Bus 13.

Table 5 – Incremental costs and nodal prices for each bus for cases 1 and 3

Case	1		3a		3b		3c		3d	
Bus	Inc. (\$/MW)	Nod. (\$/MW)								
1	3.5000	3.4495	3.5000	3.5368	3.5000	3.6119	3.5000	3.4858	3.5000	3.3480
2	3.0339	3.6004	3.3486	3.6897	3.5128	3.7667	3.2380	3.6374	2.9958	3.4953
5	3.3046	3.8283	3.3747	3.9275	3.4605	4.0104	3.3168	3.8712	3.1580	3.7192
8	3.4938	3.7652	3.4168	3.8753	3.4233	3.9622	3.4168	3.8162	3.4168	3.6582
11	3.5931	3.7542	3.8634	3.8612	3.9455	3.9477	3.8062	3.8025	3.6465	3.6456
13	3.6000	3.7047	3.8040	3.8039	3.8916	3.8929	3.7442	3.7442	3.5842	3.5844

Source: Author, 2023.

Graph 2 compares the nodal prices of wind power generation at Buses 11 and 13 in relation to the average wind speed in each season of the year. In the period from March to May (b), the region has the lowest average wind speed with 3.95 m/s and obtained the highest nodal prices, being 3.94 \$/MW at Bus 11 and 3.89 \$/MW at Bus 13. In the period from September to November (d), with the highest average wind speed, 7.79 m/s, the nodal prices were the lowest, being 3.64 \$/MW at Bus 11 and 3.58 \$/MW at Bus 13.



Source: Author, 2023

## 7 CONCLUSION

In this study, a 30-bus IEEE system was used to simulate power generation in a city in the Brazilian state of Ceará, where there is a large variation in the average wind speed throughout the year. Tests in relation to the costs of generating power to meet demand compared a thermal system with systems with both thermal and wind generators.

To compare the results of the systems mentioned, the DSOPF problem was solved with the objective of minimizing the costs of generating electricity using the MLB method. This is a

deterministic method and has the advantage over meta-heuristic methods, as it considers the dual variables (the incremental and nodal prices for each bus), enabling analysis of the incremental costs and nodal prices of the system buses.

The results showed that the replacement of thermal generators by wind generators reduces the total system costs, and through the WPDF, despite the uncertainty regarding wind speed, the wind power can be scheduled in a way that is consistent with the highest probability of occurrence in the period and in the chosen region.

Using the MLB method, showed that the nodal prices of wind generation, determined from the parameters chosen for the wind costs in the tests carried out, are practically inversely proportional to the average wind speed and, thus, the higher the average wind speed, the lower the nodal price for wind generation.

Future work involves the addition of other renewable energy sources, such as solar and biomass, and, using the MLB method, calculation of their nodal prices to compare prices between the renewable energies present in the system. This will contribute to a better analysis of the total generation cost of an electrical system considering the insertion of renewable energies into the system and their contribution to any decrease in this production cost.

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