

**Procedure for analyzing the carrying capacity of water supply: Case study  
in Bauru – SP**

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**ABSTRACT**

The objective of this paper is to develop a procedure for creating indicators of productive limits and efficiency within water supply sectors, in order to verify the demand intensity that a given population places on the availability of water production in the environment. The methodology involved collecting data through a case study in the water supply sectors of Bauru, São Paulo, by gathering various documents, records, consumption data, flow rates, and other relevant information provided by the responsible municipal authority. The relevance of the study is highlighted by the fact that the main urban centers of the country are supported by only a fraction of the available water resources due to geographical conditions. Therefore, the search for methodologies to establish efficiency indicators for the use and availability of water production is extremely important for integrated and continuous management of available resources. The results indicate that the developed model produced consistent outputs both with the observed data and the compiled verification data, thus proving sufficiently adequate to represent the phenomenon of water production in supply sectors. The study was able to compile data and provide indicators that offer a better understanding of the supply conditions of the case study, which could serve as a reference for scheduling and prioritizing investments in the supply system.

**KEYWORDS:** Stochastic frontier analyses. Production efficiency. Water infrastructure.

**1 INTRODUCTION**

The abundance of water resources in Brazil is widely recognized, which may lead to the mistaken perception that there are no water scarcity issues in the country, both quantitatively and qualitatively. However, it is important to consider that around 80% of this availability is concentrated in the Northern Region, in the Amazon Basin, where population density and water demands are lower (Brasil, 2020). Urban centers in Brazil rely on only a small fraction of this national water abundance, posing significant challenges for the infrastructure and management of water supply services. Despite advances in engineering in this field, there is still much to be done. Evaluating basic sanitation organizations in terms of infrastructure, management, efficiency, and economic-financial and socio-environmental sustainability is crucial, as proper management is directly related to the well-being and quality of life of the population (Loureiro, 2009).

The fundamental concept of centralized water supply infrastructure systems in urban areas dates back more than a century. Throughout this period, these systems have been continuously expanded to keep pace with population growth and societal needs, adapting to public health demands and environmental awareness. This infrastructure is characterized by its long lifespan and non-recoverable costs, making it heavily dependent on technological advancements (Hiessl et al., 2001). The water supply system is defined as a set of works, equipment, and services intended to provide potable water for domestic consumption, public services, industrial consumption, and other uses. It comprises various units, such as water sources, treatment plants, storage units, and distribution networks (Azevedo Netto et al., 1998).

An important element of supply systems is the supply sectors. The main guideline for designing supply sectors is to define the zones of influence of the primary units serving the sector (reservoirs, pumping stations, boosters, etc.). Defining a supply sector requires primarily considering the topographical conditions and the consumer profile in the area to be served. Once these parameters are established, it becomes possible to position the reservoirs, size the distribution lines, and define all the necessary accessories for the supply.

Previous research highlights that population growth in cities results in increased demand for water for municipal supply (Bradley et al., 2002; Falkenmark and Lindh, 1974; McDonald et al., 2011; Postel et al., 1996). According to Bartlett (2003) and Bhatia and Falkenmark (1993), this increase is not only due to demographic growth but also to a preference for municipal supply over other sources, such as private wells. This necessitates the creation of more complex urban water infrastructure systems to meet the growing demand (Alcott et al., 2013; Brown et al., 2009; McDonald et al., 2014). In this context, besides the challenges brought by rapid urbanization and increased water demand, there is concern over the limited investment capacity and low operational efficiency in supply systems, especially in developing countries.

In Brazil, for example, total billing losses in 2018 averaged 39% (Instituto Trata Brasil, 2020), raising concerns about the ability to maintain water supply in various regions. The new regulatory framework for basic sanitation, Law No. 14,026, dated July 15, 2020, aims to universalize sanitation services by encouraging competition, privatizing the sector, and privatizing state-owned sanitation companies. Additionally, it seeks to address severe environmental and public health issues caused by insufficient sanitation services in the country. By the end of 2033, it is expected that 99% of the population will have access to potable water and 90% will have access to sewage collection and treatment. The framework also includes plans to reduce losses and combat water wastage. To achieve these goals, the Ministry of Economy estimates that investments between R\$ 500 to 700 billion will be needed by the deadline (Gadelha et al., 2021).

The search for methods to establish efficiency indicators in the use and availability of water is fundamental for integrated and continuous management of water resources. It is important to highlight the scarcity of research in this field, especially concerning supply sectors, which limits analysis and planning. This work focused on developing a procedure to create indicators of production limits and efficiency in water supply sectors, aiming to better understand the water demand of a population and to guide policies and investments more effectively. The study was conducted in the city of Bauru, São Paulo, addressing various variables related to water availability and production, consumption, and demographics. From this, a model was developed to evaluate the efficiency and carrying capacity of the supply sectors.

## **2 METHODOLOGY**

### **2.1 Stochastic Frontier Analyses**

One of the existing tools that encompass the evaluation of efficiency and the concept of carrying capacity is the so-called Stochastic Frontier Analysis (SFA) models. There is a variety of research and studies involving this tool. Examples can be seen in Coelli et al. (1999) and Oum et al. (2008).

In SFA-based models, the results are not solely dependent on input variables because random components are included in the calculations. By accounting for these effects, SFA models assess chance events that impact outcomes and are not predicted by deterministic models. The theoretical definition of a production function expressing the maximum possible output value from specific input packages with fixed technology has been accepted for many

decades. Concurrently, econometrics scholars have been estimating production functions for nearly the same period. It was only since the pioneering work of Farrell (1957) that serious considerations were made about the feasibility of estimating so-called production frontier functions, in an effort to bridge the gap between theory and empirical work.

Aigner et al. (1977) and Meeusen and van den Broeck (1977) improved upon issues associated with deterministic and probabilistic production frontiers by defining the stochastic production frontier model. In this specification, the output of each firm is bounded above by the frontier, which is stochastic in the sense that its location is allowed to randomly vary across firms. Economically, this technique allows firms to be technically inefficient relative to their own frontier, unlike some sample norm. Variations between firm-specific frontiers presumably capture the effects of exogenous shocks, both favorable and unfavorable, beyond firm control. Measurement errors and output measurement issues constitute another source of frontier variation (Schmidt and Lovell, 1979).

In simplified terms, the SFA model applies the logarithmic form of the Cobb-Douglas function (Meeusen and Van Den Broeck, 1977; Coelli et al., 1999), expressed in equation (1).

$$\log y_i = \beta_0 + \sum_{i=1}^N (\beta_n \cdot \log x_i) + v_i - u_i \quad (1)$$

Where:

$y_i$  = output/production frontier variable

$x_i$  = input variables

$\beta_0, \beta_n$  = parameters estimated by the model

$v_i$  = random error term

$u_i$  = technical inefficiency

## 2.2 Case study

To achieve the proposed objectives, data collection and analysis were conducted through a case study in the water supply sectors of the city of Bauru, São Paulo state. Data collection and field research primarily involved observation, as well as gathering various documents, registries, consumption and flow data, and other relevant materials provided by the municipal authority responsible for water supply in the city. During the information analysis stage, these collected data were comparatively analyzed, tabulated, and formatted to serve as inputs for the proposed modeling.

The case study encompassed all existing water supply sectors in the city, covering an area of approximately 104 km<sup>2</sup>. It is located approximately between latitudes 22°15' S and 22°24' S and longitudes 48°58' W and 49°08' W, with altitudes ranging between approximately 490 and 600 meters. According to the latest Census, the municipality covers an area of 667,684 km<sup>2</sup> and has a population of 379,146 inhabitants (Brazilian Institute of Geography And Statistics, 2022).

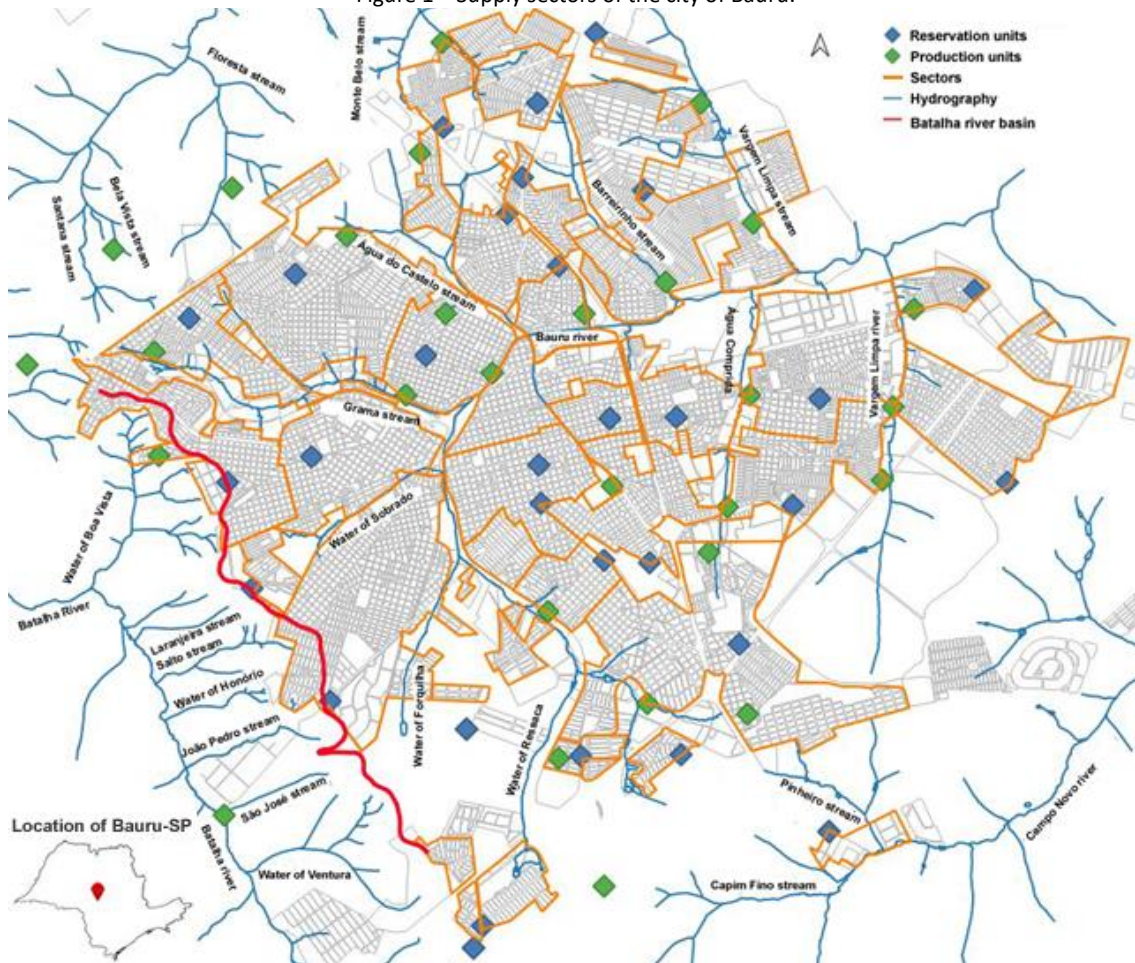
In principle, the study focuses exclusively on the urbanized areas of the municipality served by public water supply systems, excluding regions with self-supply solutions, irregular/ clandestine settlements, precarious occupations, among others. The majority of the effectively occupied urbanized area lies within the watershed of the Bauru River, except for parts of the north and south regions of the city, which are within the watershed of Água Parada stream

and Campo Novo stream, respectively. Additionally, small sections in the far west of the city are within the watershed of Batalha River.

Regarding groundwater and hydro stratigraphy in the municipality, it includes the Bauru Aquifer System (SAB), the Serra Geral Aquifer (ASG), and the Guarani Aquifer System (SAG), with the Passa Dois Aquiclude (APD) marking the basal limit of this sequence. The Bauru Aquifer System, which is granular in nature, extends throughout the municipality, covering an area of 667 km<sup>2</sup> and reaching maximum thicknesses of up to 200 meters in higher topographic regions.

The supply sectors, depicted in Figure 1, are mostly located within the Bauru River basin. The majority of water supply is provided through groundwater from various deep wells in the Guarani Aquifer, with part of the western region of the city supplied by surface water from the Batalha River. Subsequently, this water is stored in units containing reservoirs of various types and volumes, and then distributed to the population through approximately 1,700 km of pipelines made of various materials and diameters.

Figure 1 – Supply sectors of the city of Bauru.



Source: Own elaboration.

### 2.3 Tools, data used and variables employed

To perform the mathematical operations stemming from SFA modeling, the nLogit v.5 software developed by Econometric Software, Inc. and the Frontier 4.1 software developed by researcher Timothy Coelli were utilized. Additionally, the Geographic Information System (GIS) software QGIS v. 3 from the Free Software Foundation, Inc., which is freely licensed, was used to manipulate data and results, as well as to create thematic maps.

The data regarding the supply systems were collected from the Department of Water and Sewage (DAE) of Bauru, the municipal authority responsible for water supply, sewage collection, and treatment in the city. Additional data, such as the loss index in the municipal distribution system, were obtained through the National System of Sanitation Information (SNIS).

It is important to note that some of these data have variabilities that are not effectively reflected in the tabulated values. For instance, production data from both deep wells and the Batalha River water treatment plant are estimated using hour meters, which multiply the operating hours of a particular unit by its average production flow rate. This method does not accurately capture variations in flow rate and other specificities that would be properly accounted for, if flow meters and volume totalizers were installed in all units. Additionally, only a few reservoirs have flow meters for output. Moreover, there are numerous events of import and export between different sectors (such as adjustments in main pipelines and pumping between reservoirs) that occur based on demand requirements and maintenance activities but are not recorded by measuring equipment.

Another point to highlight is that the supply sectors are delineated in a way that does not entirely reflect the field reality. Few regions are truly segmented into sectors, meaning there isn't always a clear division between the networks of one sector and another. This inconsistency makes it challenging, for example, to conduct entirely accurate water balances, as there may be instances of water import and export between the networks of different sectors based on specific consumption demands and other situations where there is inadequate capacity for recording and measuring these transactions.

Given these considerations, the possible variables to be used in estimating the SFA model are presented as follows:

- Average available flow ( $Q_i$ ): Dependent variable of the quantity produced, calculated considering nominal production flows, import and export events between sectors, and existing system losses, in cubic meters per hour ( $m^3/h$ ).
- Territory ( $A_i$ ): Total territorial area served by the supply sector, in square kilometers ( $km^2$ ).
- Supply networks ( $L_i$ ): Total linear extent of networks of various diameters within each supply sector's territory, responsible for transporting the produced flow between units and distributing it to the end consumer, in kilometers (km).
- Economies ( $N_i$ ): Total number of consumer units within the supply sector, in units.
- Energy consumption ( $E_i$ ): Energy consumption used in each sector, considering consumption related to deep wells, raw water pumping pumps, boosters, and various pumping units for proper supply, in kilowatt-hours (kWh).

- Chemical consumption ( $P_i$ ): Consumption of various chemicals (aluminum sulfate, lime, chlorine, fluosilicic acid, polyphosphate, chlorine dioxide, sulfuric acid, sodium hydroxide, and sodium hypochlorite) used in the treatment of produced water, both at the raw water treatment plant and in deep well treatment units, in kilograms (kg).
- Population density ( $D_i$ ): Ratio of estimated inhabitants for the sector to the total territorial area covered by it, in inhabitants per square kilometer (inhabitants/km<sup>2</sup>).
- Average income ( $R_i$ ): Average income composition for each supply sector, obtained through IBGE data, in current monetary units (R\$).

For the definition of the average available production flow ( $Q_i$ ), the following procedures were used. Analogous definitions were used for defining energy consumption and chemical consumption when necessary.

- Units supplied by production reservoirs were defined;
- The captured and treated production at the water treatment plant, with the system in full operation, currently averages 1,980.00 m<sup>3</sup>/h (despite having a granted flow rate of 1,252.80 m<sup>3</sup>/h) from the Rio Batalha surface water source. This production directly supplies three different storage units: UR00 - ETA, UR01 - Praça Portugal, and UR05 - Alto Paraíso. Since these reservoirs do not have flow meters at their water inlets, a percentage of the total production flow was considered to be directed to each based on historical analysis. This percentage was proportionally defined according to the number of consumer units (economies) existing in each of the sectors;
- For production units from wells that, through daily maneuvers, supply two different storage units and due to the absence of macro meters installed, it was considered that each reservoir receives half of the production. This decision was made because the historical data on these maneuvers shows significant variation due to specific demands. This situation applies to the following production units: UP18 - Beija Flor, UP27 - Cruzeiro do Sul, UP32 - Nicéia, UP36 - Samambaia, UP49 - Cardia, and UP52 - Bauru XVI III;
- Production flows from deep wells were reduced by approximately 0.833 compared to nominal flows, as their maximum pumping periods are limited to 20 hours per day (DAEE, 2018);
- Import and export events between different reservoirs (through pumping or advection) were considered using the nominal flow rates of existing pumps or boosters, as well as the advection flow rates through the Hazen-Williams formula. For these events, the following were considered: pumping from UR23 - Gasparini to UR12 - IX de Julho; pumping from UR05 - Alto Paraíso to UR15 - Vila Seca; pumping from UR00 - ETA to UR34 - Sabiás; advection from UR00 - ETA to UR01 - Praça Portugal; advection from UR00 - ETA to UR05 - Alto Paraíso; advection from UR01 - Praça Portugal to UR02 - Sede; advection from UR05 - Alto Paraíso to UR03 - Bela Vista; pumping from UR19 - Redentor to UR07 - Geisel; pumping from UR08 - Jasmins to UR06 - Ipê; and pumping from UR37 - Zona Norte to UR44 - LEB;
- Finally, the current total loss index in distribution in the city of Bauru-SP was applied, which is 46.50% of the produced volume, obtained through the IN049 indicator from SNIS (BRASIL, 2020).

Finally, to estimate the population density of the sector, the relationship between the total population at the time of data processing, estimated at 381,706 people (Brazilian Institute of Geography and Statistics, 2021), and the total number of households, which was 182,520 at the same time, was defined as 2.09 inhabitants per household. This value was then multiplied by the number of households registered in each sector.

## 2.4 Model development and calibration

The model to be used, if composed of all possible variables listed, follows the original specification of Battese and Coelli (1995), using the natural logarithm. The data used correspond to only one observation period (June 2021 reference), therefore the model can be considered as a special case of the general model with time factor set to unity. Using the natural logarithm of the Cobb-Douglas form, the variables of interest are represented by their natural logarithm in Table 1.

Table 1 – Possible model variables represented by their natural logarithm.

ID	Sector	ln(Q <sub>i</sub> )	ln(P <sub>i</sub> )	ln(E <sub>i</sub> )	ln(L <sub>i</sub> )	ln(N <sub>i</sub> )	ln(A <sub>i</sub> )	ln(R <sub>i</sub> )	ln(D <sub>i</sub> )
1	ETA	5,50	11,14	12,28	5,07	9,78	2,11	7,61	8,40
2	Praça Portugal	5,51	10,49	12,69	4,34	9,47	1,13	8,43	9,08
3	Sede	5,54	11,19	12,14	3,79	9,04	0,88	7,58	8,90
4	Bela Vista	5,43	10,69	12,30	4,20	9,09	1,02	7,38	8,81
5	Parque Paulistano	4,22	6,96	11,70	3,68	8,44	0,72	7,59	8,45
6	Alto Paraíso	4,76	10,62	12,14	4,40	9,30	1,31	7,68	8,72
7	Ipê	4,90	7,77	11,77	3,70	8,71	1,00	7,32	8,45
8	Geisel	5,32	8,33	12,40	4,27	9,04	1,08	7,30	8,70
9	Jasmins	4,32	7,11	11,22	3,41	8,21	0,41	7,35	8,54
10	Jardim América	4,35	7,04	11,54	3,41	8,01	0,28	8,76	8,47
11	IX de Julho	5,31	8,22	12,83	4,91	9,67	1,82	7,13	8,59
12	Vila Seca	5,22	10,19	12,31	4,54	9,13	1,38	7,14	8,49
13	Octávio Rasi	4,58	6,09	10,46	2,97	7,78	-0,24	7,30	8,76
14	Redentor	4,35	7,34	12,24	4,32	8,94	1,58	7,25	8,10
15	Tibiriçá	2,36	5,36	8,50	1,94	6,02	-1,09	7,30	7,85
16	Gasparini	4,26	6,95	11,78	3,73	8,29	0,48	7,11	8,55
17	Mary Dota	4,95	8,19	12,28	4,65	9,35	1,62	7,24	8,46
18	Shopping	4,88	6,67	12,01	4,26	9,53	1,07	8,37	9,20
19	Nova Esperança	4,88	8,16	12,20	3,97	8,80	0,88	7,16	8,66
20	Sabiás	4,54	10,19	9,90	2,72	7,45	-0,35	7,30	8,53
21	Santos Dummont	4,59	7,46	12,24	4,69	8,79	2,14	9,04	7,39
22	Manchester	4,23	7,23	11,48	3,84	7,86	1,54	7,04	7,06
23	Zona Norte	5,13	8,21	12,83	4,04	8,73	0,96	7,09	8,51
24	Villagio	3,12	5,70	9,91	2,40	6,13	-0,69	9,58	7,56
25	Chácaras Bauruenses	2,09	3,91	8,28	1,50	3,26	-0,37	8,52	4,37
26	Lago Sul	3,04	5,83	9,74	2,88	6,19	0,04	9,58	6,88
27	Cardia	3,81	6,70	11,17	3,06	8,09	-0,06	7,86	8,89
28	Imperial	3,97	5,95	11,00	3,40	6,90	0,45	9,36	7,19
29	Alphaville	3,23	3,18	8,24	2,51	4,06	-0,74	9,58	5,54
30	LEB	3,57	6,65	9,44	3,39	8,36	1,20	6,95	7,89
31	Estoril Premium	3,87	2,56	8,18	1,79	3,30	-1,43	9,26	5,46

Source: Own elaboration.



Next, in order to assess potential redundancies among the listed variables, linear correlation was examined between the variables presented, using the Pearson correlation coefficient. Significant linear correlations, exceeding 90%, were observed between the variables of energy consumption and supply networks, energy consumption and economies, supply networks and economies, and supply networks and territory. Additionally, the supply networks variable was present in three out of the four cases analyzed. Considering its considerable linear correlation with other variables in the model, and to avoid potential biases, it was excluded from the proposed model.

Furthermore, the demographic density variable is a ratio expressed between the economies and territory variables multiplied by a factor representing the average number of inhabitants per economy. Therefore, to avoid potential biases, only the demographic density variable was used. Additionally, a significant linear correlation was also identified between the available average flow rate, electricity consumption, and chemical consumption. Hence, all these variables are significant enough to potentially be dependent variables in the model to be constructed. In practical terms, these variables constitute the productive output of a supply sector (dependent variables), while demographic density and average income variables constitute the sector's demand (independent variables). Thus, models were developed with various combinations of the listed variables, aiming to find the combination that best describes the phenomenon to be represented.

### 3 RESULTS

To evaluate the proposed models, their respective values of the Akaike Information Criterion (AIC) were compared, which assesses the goodness of fit of the parametric model estimated by maximum likelihood (Moura, 2021). The models were estimated using the nLogit software, and the results regarding the significance of the variables and the AIC values are presented in Table 2.

Analyzing the calculated AIC values for the different models, it is observed that the model with the lowest value was model ID (2), represented below by equation (2), comprising the dependent variable average production flow ( $Q_i$ ) and the independent variable population density ( $D_i$  – significant at the 1% level), along with the constant. Therefore, this model was chosen to describe the phenomenon in question.

$$\ln Q_i = \beta_0 + \beta_1 \cdot \ln D_i + v_i - u_i \quad (2)$$

Where:

$Q_i$  = output/production frontier variable;

$D_i$  = input variable;

$\beta_0$  e  $\beta_1$  = parameters estimated by the model;

$v_i$  = random error term;

$u_i$  = technical inefficiency.

Table 2 – Analysis of AIC for the proposed models.

ID	AIC	Dependent variable	Independent variables		
			Constant	Density	Income
1	43,90	Flow rate	---	0,52700***	0,10749
2	43,80		1,32467	0,46669***	---
3	45,80		1,22200	0,47137***	0,00833
4	80,60	Electricity	---	1,18772***	0,42727*
5	77,00		5,18584***	0,87762***	---
6	78,90		5,84132	0,86263***	-0,07650
7	97,70	Chemicals	---	1,21642***	-0,26229
8	---		---	---	---
9	---		---	---	---

Legend: \*\*\*, \*\* e \* - Significance at the level of 1%, 5% e 10%, respectively.

Source: Own elaboration.

It is worth noting that for the development of the model considering the 31 sectors analyzed, a proportion of 80% of them were allocated to calibration (25 sectors), while the remaining 20% were reserved for model verification and validation (6 sectors). Additionally, in the proposed model, two estimates were conducted: one assuming that the error term capturing inefficiency follows a truncated normal distribution, and another assuming a half-normal distribution. The model results indicated that the half-normal assumption is not appropriate, thus the analysis focused on the estimation with a truncated normal distribution for the error term. The coefficients returned are summarized in Table 3.

Table 3 – Coefficients calculated for the adopted model.

Variables and parameters	Coefficient	Standard error	z	Prob.  z >z	Confidence interval 95%
Constant $\beta_0$	1,32467	0,81099	1,63	0,1024	-0,26484 2,91418
Demographic density $\beta_1$	0,46669	0,09276	5,03	0,000	-0,28488 0,64850
$\lambda$	57723,4	0,1349e <sup>9</sup>	0,00	0,9997	---
$\sigma$	1,30515	0,85881	1,52	0,1286	-0,37809 2,98839

Source: Own elaboration.

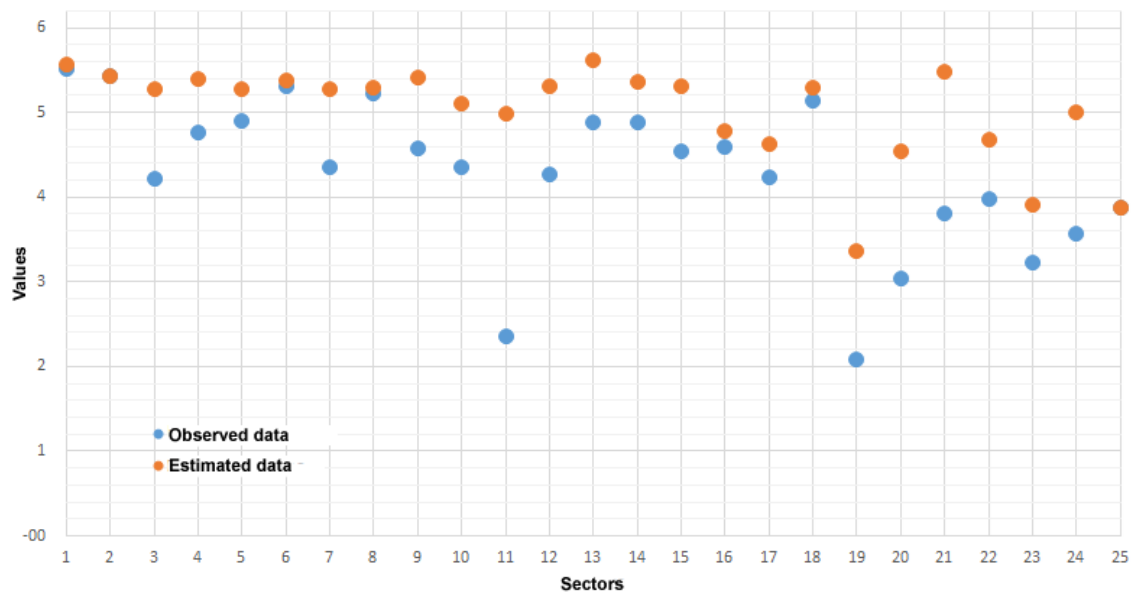
The results show a good fit of the Cobb-Douglas production function, given that the main estimated parameter of population density is significant at the 1% level. The findings are consistent both in terms of expected signs and the importance of each parameter in constructing the model. The comparison between the observed data and the values calculated by the model, for both the calibration and verification sectors, is presented in Tables 4 and 5, as well as Figures 2 and 3.

Table 4 – Comparison of the observed data and model-calculated values during calibration.

Sector	Observed data	Model		
		Calculated	Variation	%
1	5,5128	5,5622	-0,0494	-0,90
2	5,4341	5,4362	-0,0021	-0,04
3	4,2173	5,2682	-1,0509	-24,92
4	4,7596	5,3942	-0,6346	-13,33
5	4,8966	5,2682	-0,3716	-7,59
6	5,3166	5,3849	-0,0683	-1,28
7	4,3522	5,2775	-0,9253	-21,26
8	5,2227	5,2869	-0,0642	-1,23
9	4,5808	5,4129	-0,8321	-18,16
10	4,3523	5,1049	-0,7526	-17,29
11	2,3552	4,9882	-2,6330	-111,79
12	4,2625	5,3149	-1,0524	-24,69
13	4,8783	5,6182	-0,7399	-15,17
14	4,8750	5,3662	-0,4912	-10,08
15	4,5389	5,3055	-0,7666	-16,89
16	4,5910	4,7735	-0,1825	-3,98
17	4,2262	4,6195	-0,3933	-9,31
18	5,1324	5,2962	-0,1638	-3,19
19	2,0869	3,3641	-1,2772	-61,20
20	3,0445	4,5355	-1,4910	-48,97
21	3,8118	5,4735	-1,6617	-43,59
22	3,9701	4,6802	-0,7101	-17,89
23	3,2253	3,9101	-0,6848	-21,23
24	3,5737	5,0069	-1,4332	-40,10
25	3,8722	3,8728	-0,0006	-0,02

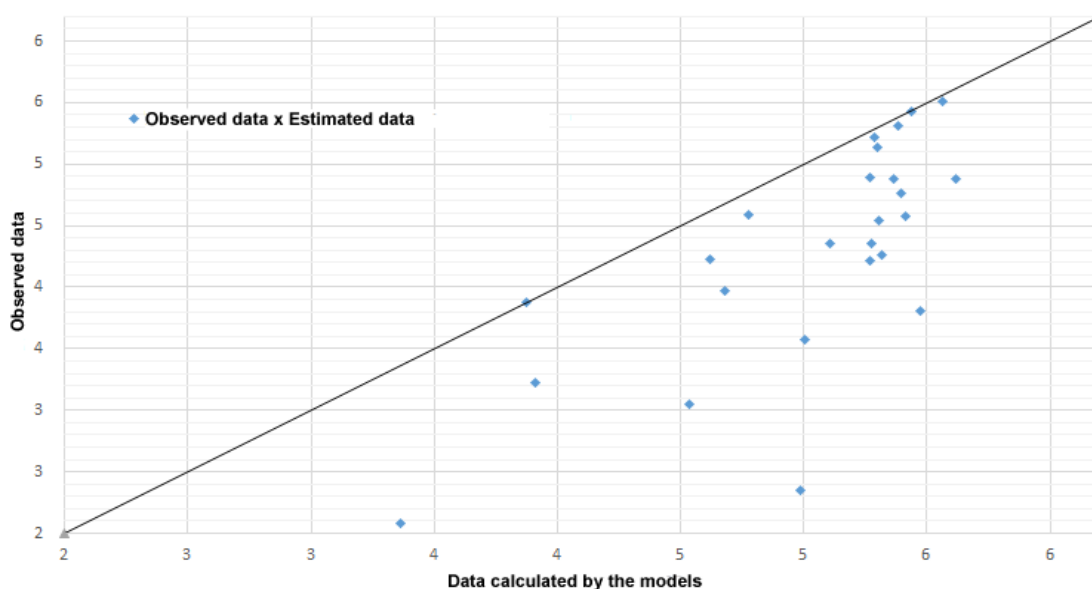
Source: Own elaboration.

Figure 2 – Values of observed data and model-calculated values during calibration.



Source: Own elaboration.

Figure 3 – Comparison of observed data and model-calculated values during calibration.



Source: Own elaboration.

Table 5 – Comparison of observed data and model-calculated values during validation.

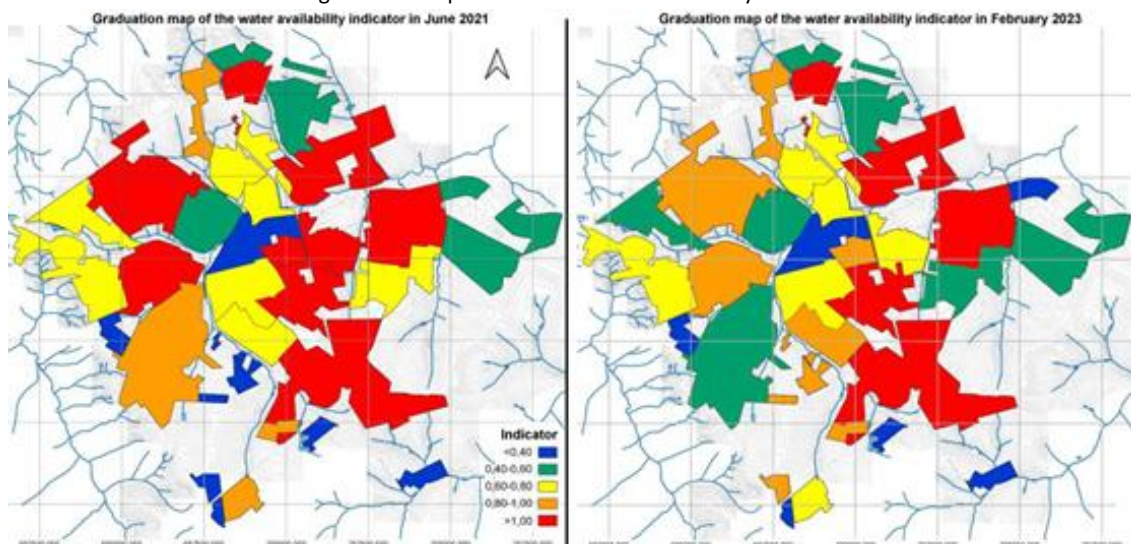
Sector	Observed data	Model		
		Calculated	Variation	%
1	5,5010	5,2449	0,2561	4,66
2	5,5435	5,4782	0,0653	1,18
3	4,3239	5,3102	-0,9863	-22,81
4	5,3112	5,3335	-0,0223	-0,42
5	4,9539	5,2729	-0,3190	-6,44

Source: Own elaboration.

These data corroborate that the model demonstrates sufficient adequacy to represent the phenomenon, as in most cases the calculated data closely approximate the observed values, both in the calibration and model construction sectors as well as in the verification sectors.

Additionally, as part of the production frontier analyses, one specific objective of the study was to develop a database containing the results of the calculation of water availability for the existing supply sectors in the municipality of Bauru, based on compiled and developed data throughout the case study. To compose this analysis, the already developed and estimated data of average production flow and average consumption per sector were considered, both transformed into cubic meters per hour. This allowed the construction of an adimensional indicator of water availability for each of the 31 sectors, as shown in Figure 4, based on references from June 2021 and February 2023, reflecting the outcomes of interventions and works carried out by the authority responsible for water supply to enhance the system.

Figure 4 – Comparison of the water availability indicator.



Source: Own elaboration.

The changes observed in the calculated indicators demonstrate progress in water availability in some sectors affected by the mentioned works. These changes directly impact the region supplied by the Batalha River, which experiences high seasonal variability in water availability and significant operational challenges during dry periods. Currently, there are situations where water is extracted from the Batalha River at a flow rate of 1,368.00 m<sup>3</sup>/h, significantly lower than the initially considered 1,980.00 m<sup>3</sup>/h (according to the June 2021 reference) and close to the permitted flow rate of 1,252.80 m<sup>3</sup>/h. This allows for a less predatory exploitation of the water source and greater capacity to sustain supply during dry periods without interruptions for the population.

#### 4 CONCLUSION

The results of this study achieved the established objectives: estimating a stochastic frontier production function for water supply sectors and developing a database with the results of calculating water availability for the existing supply sectors in the municipality of Bauru. Additionally, it provided a method for analyzing water supply carrying capacity using supply sectors.

The stochastic frontier production function, which considered average production flow of a sector as the dependent variable and population density as the independent variable, yielded statistically significant results both in sectors used to calibrate the model and those used for verification. However, during the Bauru case study, some limitations in the data were identified, which could be revisited in the future to improve the developed model. Uncertainties related to exact water production values, import and export events between sectors, specific water losses in each unit, and the physical and geographical delineation of sectors are aspects to be considered for future model enhancements.

Replicating and expanding the case study procedure considering, for example, data from other cities with more precisely measured variables, are future possibilities for continuity and

improvement in the focus of this work. However, it is important to note that what can be replicated is the modeling procedure and not necessarily the model developed in this study. This is due to empirical nature and the types of available data. Therefore, any expansion of studies through replication of the procedure should include careful verification of the time frame and data considered, ensuring that the procedure and new models developed are statistically representative for new cases.

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