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# Cost optimization considering the probability of failure in reinforced concrete beams in bending.

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**ABSTRACT**

This paper aims to demonstrate the feasibility and relevance of applying structural optimization methods together with the probability of failure constraints, integrating the search for the optimum solution and the guarantee of structural safety. The methodology involved optimization methods such as sequential quadratic (SQP), interior-point, and active-set, along with the FORM probability of failure calculation method. Computational tools such as MATLAB enabled a comparative analysis of the performance of these methods. The study stands out for integrating structural optimization and reliability, applying it to reinforced concrete beams, a relevant topic for structural engineering. The results of the analysis of simply supported beams showed satisfactory convergence between the methods, with minimal variability and superior performance of the SQP and interior-point optimizers. The introduction of standardized safety coefficients increased the reinforcement rate, reducing the probability of failure and costs. The study highlighted the synergy between optimization and structural safety, contributing theoretically by showing the effectiveness of combining optimization with probability of failure restrictions, and methodologically by comparatively evaluating different optimization methods.

**KEYWORDS:** Optimization. Probability of failure. Reinforced concrete beams.

## 1 INTRODUCTION

When carrying out a project, structural engineers must always seek results that meet both the economy and safety of the structure. Optimization methods are very relevant in the search for economy, as they minimize the cost function (objective function) subject to restrictions that are prescribed in design standards. The literature and programming language libraries, such as MATLAB (which will be used in this work), are well-established in the application of these methods to structures, facilitating their use and analysis.

Reliability methods are very useful when seeking safety, since the quantities involved have measurement uncertainties, leading to statistical modeling that shows potential variability, uncertainties that arise mainly in concrete production and project execution. The Brazilian standard itself recognizes these uncertainties, so much so that it establishes safety coefficients that must be applied in the calculations to ensure greater safety in the process.

Motta and Afonso (2016) analyze the application of optimization methods linked to reliability analysis in spatial truss analysis, with robust geometry in their modeling and satisfactory results. Motta et al (2021) also applied reliability analysis methods to the analysis of corroded pipelines, which are issues involving significant uncertainties and non-linearities in the process. Optimization methods are also widely used in fluid mechanics, as Horowitz (2013) showed for the analysis of the operation of oil reservoirs.

This study aims to optimize the cost of a simply supported beam subjected to bending stresses, with the probability of failure (obtained by reliability methods) as one of the constraints of this process. In short, the aim is to reduce costs with safety as one of the constraints. As a result, a comparison can be drawn between the methods applied and discussions can be held based on these results. In the case of reinforced concrete structures, the calculation models, procedures, and restrictions are established according to the ABNT NBR 6118:2023 standard, thus being the basis for this work.

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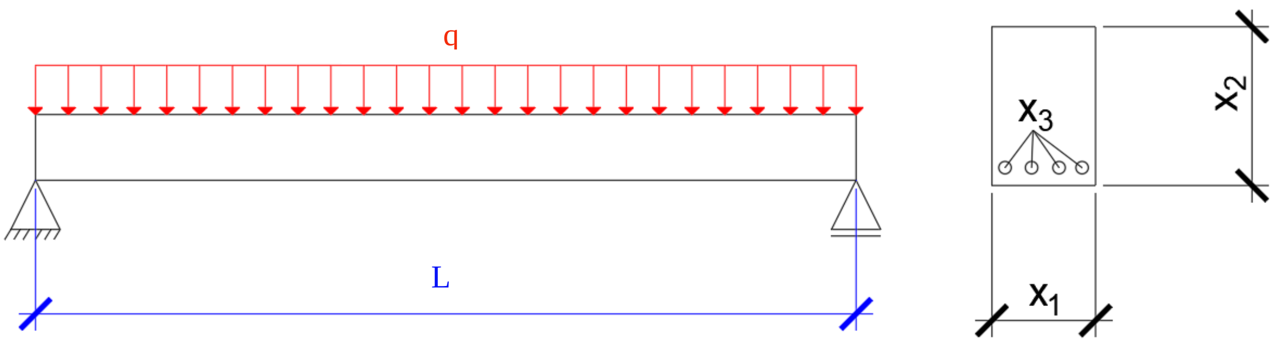
## 2 METHODOLOGY

The problem to be solved is cost optimization (concrete, formwork, steel) of a simply supported beam, with a constant load (q) distributed along its span (L). As shown in Figure 1, the design variables are the dimensions of the cross-section: width (x1), height (x2, and steel area (x3).

Based on the National System for Research on Costs and Indices of Civil Construction (SINAPI), we could build the cost function to be optimized, therefore called the objective function. Table 1 shows the compositions used and their measurement units, taking the cost values for the state of Pernambuco for March 2024. It is therefore easy to obtain the objective function, as shown in Equation 1, where ρ is the specific weight of the steel.

Figure 1 – Simply supported beam




Source: Elaborated by the authors, 2023.

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Table 1 – Unit costs

| Code | Description | Unit | Cost (R$) |
|---|---|---|---|
| 96557 | Concreting | M3 | 654.30 |
| 96530 | Form assembly | M2 | 156.96 |
| 104920 | Reinforcement | KG | 11.91 |

Source: SINAPI, 2024.

$$cost = cost_{concrete} * x_1 x_2 L + cost_{form} * (x_1 L + 2x_2 L) + cost_{beam} * x_3 L\rho \quad \text{Eq. 1}$$

This problem deals with bending and the maximum deflection of the beam. Bending occurs when two equal and opposite bending moments act in the same longitudinal plane (BEER et al, 2015), resulting in the formation of two regions of stress in the cross-section: tension and normal compression. Concrete has high compressive strength, but its fragility and low tensile strength restrict its use in isolation. To overcome these limitations, steel is used jointly with concrete, conveniently positioned in the part to resist tensile stresses (BASTOS, 2019).

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The maximum deflection is the main deformation that occurs on the elastic line, which is the bending diagram of the longitudinal axis that passes through the centroid of each area of the beam's cross-section (HIBBELER, 2010). Therefore, the concepts of bending and flection are interlinked and present important normative constraints to be dealt with.

By considering bending stresses, the optimization process requires balancing the resistive moment, where fyk is the characteristic yield strength of the steel, fck of the concrete, and c the cover of the cross-section (Equation 2), against the requesting moment (Equation 3), where the resistive moment represents the capacity of the cross-section to withstand bending, while the requesting moment is the stress present in the structure under analysis. It is worth noting that such a balance is achieved considering the safety coefficients established in the standard in item 12.4.1; however, the equations presented consider this balance based on the variables of the problem.

$$M_R = f_{yk}x_3\left(x_2 - c - \frac{0.588 f_{yk}A_s}{x_1 f_{ck}}\right) \quad \text{Eq. 2}$$

$$M_S = \frac{qL^2}{8} \quad \text{Eq. 3}$$

The reinforcement required to withstand the tensile forces is obtained from the balance between these moments. ABNT NBR 6118:2023, in its items 17.3.5.2.1 and 17.3.5.2.4, establishes, respectively, restrictions for the minimum reinforcement (Equation 4), in which up to a fck of 30 Mpa, the ρ_min factor is 0.15%, and the maximum (Equation 5), to be added to the section and therefore considered in the optimization process.

$$A_{min} = \rho_{min}x_1x_2 \quad \text{Eq. 4}$$

$$A_{max} = 0.04x_1x_2 \quad \text{Eq. 5}$$

The bending moment and reinforcement rate restrictions above are clearly non-linear, as they depend on the optimization process variables themselves. In addition, linear restrictions were also considered, such as the beam width restriction in item 13.2.2 of the standard, as well as height restrictions (minimum 30 cm) and reinforcement rate (minimum 1 cm²), the latter two being adopted due to practical construction issues, and therefore not required by the standard.

Finishing off the restrictions, we have the bending equation for the conditions of the problem addressed (Equation 6), where E is the modulus of elasticity, derived from solving the differential equation of the elastic line (BEER et al, 2015). Item 13.3 of NBR 6118:2023 limits the deformation to L/500 and 10 mm for the case of beams supporting walls, in addition to the rotation limit of 0.0017 rad, where such rotation occurs in simple supports, which is the case addressed. This rotation is also calculated from the elastic line (Equation 7), the first derivative of the elastic line equation being.

$$y\left(\frac{L}{2}\right) = \frac{5}{32}\frac{qL^4}{Ex_1x_2^3} \quad \text{Eq. 6}$$

$$\frac{dy}{dx}(0, L) = \frac{1}{2}\frac{qL^3}{Ex_1x_2^3} \quad \text{Eq. 7}$$

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The MATLAB language is an option for solving optimization problems, as it includes the Optimization Toolbox library with functions for finding parameters that minimize or maximize objective functions while satisfying constraints (Mathworks, 2024). To deal with non-linear problems, the library provides the fmincon function, which finds the minimum of problems with non-linear constraints with different variables (Mathworks, 2024), therefore solving the problem concerned.

The fmincon function allows the user to choose the algorithm that will be used to solve the problem, so it is worth comparing them. To this end, we chose the "interior-point" (function default), "sqp," and active-set algorithms, which are well-known methods in the optimization literature, and plotted the iterations of the methods, enabling us to draw reliable conclusions.

The variables presented in the equations previously described show variability due to uncertainties in the process of obtaining them. These uncertainties can be related to measurement errors, natural phenomena, and manufacturing errors, among others (BECK, 2008). Therefore, for a higher level of certainty in the results, it is important to consider these variabilities, using a study based on the concepts established by structural reliability.

These variables associated with uncertainties are called random variables and carry statistical information that is gathered experimentally. The information is mainly the coefficient of variation and the type of probability distribution (normal, lognormal, etc.). Such information allows the probability of failure to be calculated, which is the (subjective) probability that the system will fail, not meeting the design specifications (BECK, 2008).

Several methods are available for calculating the probability of failure. Our choice for this work is the FORM method (first-order reliability method), which consists of building a joint probability function and transforming it into a Gaussian distribution. This transformation represents a one-to-one mapping, which takes points from the original domain X to domain Y. Within the algorithm of the method, there is an optimization process during the transformation between the domains, in which the gradient is calculated and transformed from domain X to Y (BECK, 2008). This internal procedure, based on the calculation of gradients, shows a familiarity between the method and optimization procedures.

Therefore, the probability of failure restriction will be considered for the bending stresses and the deformations related to deflection and rotation, aiming to improve the safety of the results due to the uncertainties of the variables used in the equations, which can lead to failures. These uncertainties are identified based on NBR 6118:2023, item 12.4.1, which establishes safety coefficients of 1.4 and 1.15, respectively, in the calculation procedure, reducing the strengths and increasing the loads. For comparative purposes, the simulation will be carried out with and without these coefficients, allowing conclusions to be drawn about their influence on the optimization results and the reduction, as expected, of the probability of failure.

## 3 RESULTS AND DISCUSSION

In addition to Table 1, Table 2 shows the values adopted for the variables in the equations, containing values that are close to practical cases in reinforced concrete structural calculations.

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Table 2 – Values used for constants

| Variable | Description | Value | Unit |
|---|---|---|---|
| L | Beam span | 5 | m |
| q | Distributed loading | 10 | kN/m |
| fck | Compressive strength of concrete | 25000 | kN/m² |
| fyk | Yield strength of steel | 500000 | kN/m² |
| ρ | Mass of steel | 7850 | kg/m³ |
| c | Cross-section coverage | 0.025 | m |
| E | Modulus of elasticity | 30000000 | kN/m² |

Source: Elaborated by the authors, 2024.

In terms of the probability of failure, statistical data must be provided for calculating the probability of failure. We adopted the values exemplified by the Joint Committee on Structural Safety (JCCS) in their list of publications available on their website. Table 3 shows these values, whilst Table 2 shows the averages. The minimum probability of failure to be met in the structural reliability constraints was 0.01%.

Table 3 – Random variables

| Variable | Unit | Coefficient of variation | Distribution |
|---|---|---|---|
| fyk | kN/m² | 0.0536 | Lognormal |
| As | m² | 0.05 | Normal |
| h-c | m | 0.0435 | Normal |
| fck | kN/m² | 0.1833 | Lognormal |
| q | kN | 0.1 | Normal |

Source: Vrouwenvelder et al, 2012.

### 3.1 Case 1: Disregarding safety coefficients

By performing the optimization without considering the safety coefficients required by the standard, as mentioned above, the final cost obtained by each optimizer can be compared in Table 4. Despite the insignificant difference, the SQP optimizer resulted in a lower cost and the other methods had similar costs, with active-set providing the highest cost.

Table 4 – Case 1: Optimized cost

| Optimizer | Cost (R$) |
|---|---|
| SQP | 1,192.50 |
| Interior-point | 1,212.44 |
| Active-set | 1,217.63 |

Source: Elaborated by the authors, 2024.

Regarding convergence, the graph in Figure 2 shows that the SQP method, in addition to having the lowest cost, also converged more quickly than the other methods, so that the interior-point required more than twice as many iterations. The active-set showed considerably unsatisfactory convergence, taking almost 35 iterations in total. We also noticed high peaks around the fifteenth and thirtieth iterations, a peak which then declined. We can also see that after the fifteenth iteration until the thirtieth, there was very little variation between the results, but without the process being terminated, which led to an unnecessary computational cost, ending up with the highest cost of the three methods, as shown below.

Table 5 shows the results of the optimization variables described in Figure 1 for each optimizer. Despite the lower costs, the cross-section resulting from the SQP method had a larger

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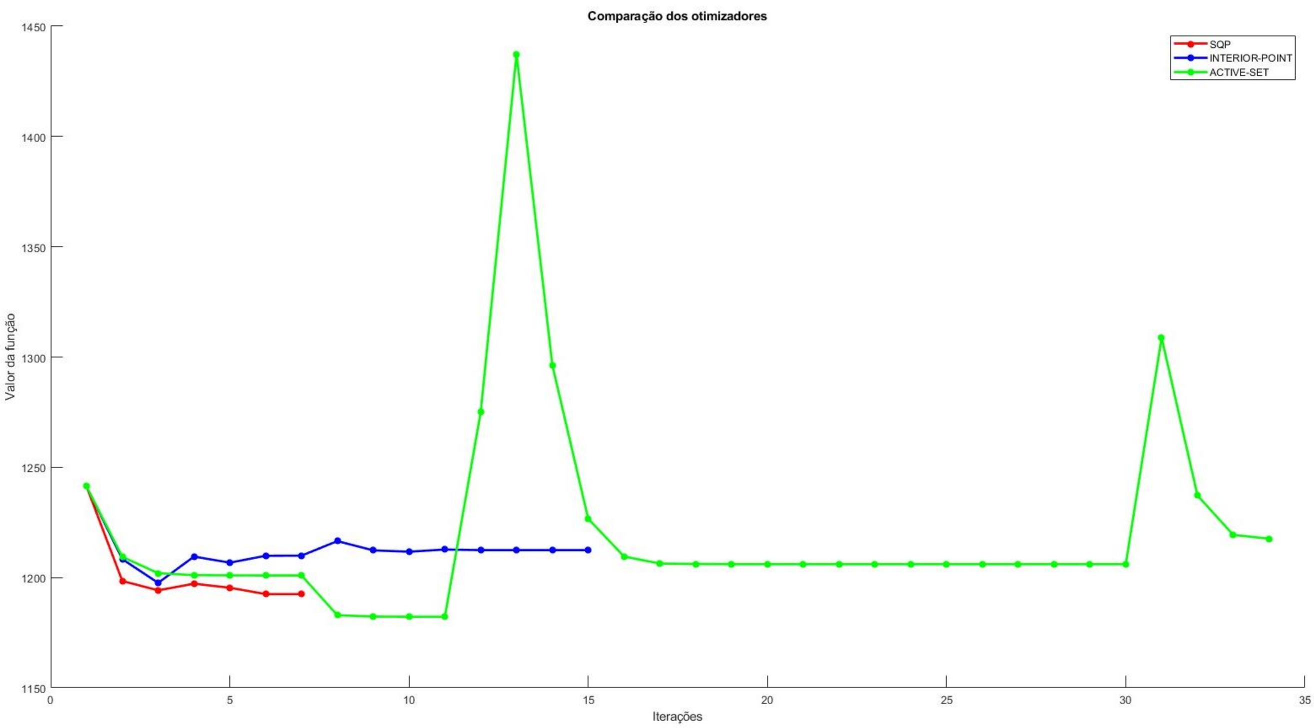
cross-sectional area, but a reduced steel area compared to the other methods, which contributed to the lower cost. The active-set resulted in the minimum base assigned among the constraints; however, it ended up compensating in height, having the largest dimension. The interior-point resulted in the largest steel area, although with the second lowest cost.

Table 5 – Case 1: Result of the project variables

| Optimizer | Width (x1) [cm] | Height (x2) [cm] | Steel area (x3) [cm²] |
|---|---|---|---|
| SQP | 13.69 | 49 | 2.06 |
| Interior-point | 13.23 | 50.3 | 2.17 |
| Active-set | 12 | 52.21 | 2.11 |

Source: Elaborated by the authors, 2024.

Figure 2 – Case 1: Convergence comparison




Source: Elaborated by the authors, 2024.

In terms of execution, the dimensions presented are not adequate, therefore Table 6 provides the results closer to practical reality, rounding off the dimensions of the cross-section and the number of steel bars and their diameter. Given this organization, we see that the reinforcements end up being equal between the methods so that two 12.5 mm diameter bars result in a steel area of 2.44 cm², which is more advantageous in terms of execution than using three 10 mm bars, with a steel area of 2.36 cm².

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Table 6 – Case 1: Variables in terms of execution

| Optimizer | Width (x1) [cm] | Height (x2) [cm] | Reinforcement |
|---|---|---|---|
| SQP | 14 | 49 | 2φ12.5 |
| Interior-point | 14 | 51 | 2φ12.5 |
| Active-set | 12 | 53 | 2φ12.5 |

Source: Elaborated by the authors, 2024.

### 3.2 Case 2: Considering the safety coefficients

By adding the safety coefficients required by ABNT NBR 6118:2023, as mentioned above, Table 7 shows the result of the optimized cost, as well as the increase in cost compared to case 1. Once again, the SQP optimizer resulted in the lowest cost and the smallest increase compared to the case without the safety coefficients, just as the active-set resulted in the highest cost and the greatest difference. The increase in costs was to be expected since the safety coefficients are applied to increase the loads and decrease the resistive stresses, thus requiring higher costs to meet the constraints.

Table 7 – Case 2: Optimized cost

| Optimizer | Cost (R$) | Increase over case 1 (R$) |
|---|---|---|
| SQP | 1,239.91 | 47.41 |
| Interior-point | 1,294.89 | 82.45 |
| Active-set | 1,362.72 | 145.09 |

Source: Elaborated by the authors, 2024.

Comparing the convergence of the optimizers (Figure 3), the active-set method remained with a high computational cost without a variation in the function value that justified the high number of iterations. The difference came when compared to the other methods, with the interior-point method requiring one less iteration than the SQP, even though the cost was higher.

The results of the design variables (Table 8) show that, as already concluded from the costs, the cross-section increased compared to case 1 and the steel area by around 1 cm². Once again, the active-set delivered more robust results, although the thickness of the cross-section once again resulted in the minimum established in the constraints.

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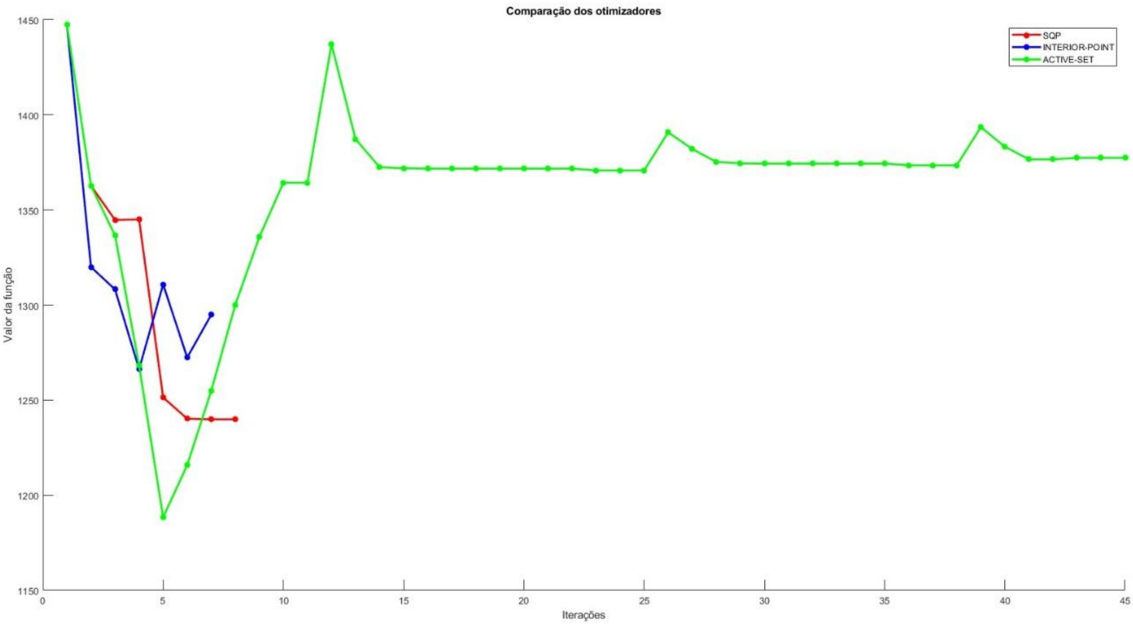
Figure 3 – Case 2: Convergence comparison




Source: Elaborated by the authors, 2024

Table 8 – Case 2: Result of the project variables

| Optimizer | Width (x1) [cm] | Height (x2) [cm] | Steel area (x3) [cm²] |
|---|---|---|---|
| SQP | 16.83 | 44.72 | 3.41 |
| Interior-point | 12.96 | 51.64 | 3.5 |
| Active-set | 12 | 55.14 | 3.99 |

Source: Elaborated by the authors, 2024

Regarding execution (Table 2), as long as only two bars are used, it is clear that the 16 mm diameter must be used, unlike the 12.5 mm diameter used in case 1. It is worth noting that if three 12.5 mm bars (3.66 cm²) were used, the results of the SQP and interior-point optimizers would be met, but not the active-set, which delivered more robust results than the previous case.

Table 9 – Case 2: Variables in terms of execution

| Optimizer | Width (x1) [cm] | Height (x2) [cm] | Reinforcement |
|---|---|---|---|
| SQP | 17 | 45 | 2φ16 |
| Interior-point | 13 | 52 | 2φ16 |
| Active-set | 12 | 56 | 2φ16 |

Source: Elaborated by the authors, 2024

As explained above, safety coefficients are required to ensure greater confidence in engineering projects, being drawn from the concepts of structural reliability. A comparison of the maximum probability of failure calculated by the FORM method and achieved by the results of each optimizer in both cases (Table 10) shows that adding the coefficients considerably reduces the probability of failure, even for the sqp method, which was the most economical. The large difference in the results of the active-set method indicates that the method was oversized so that the minimum probability of failure to be met was in the order of 10-4 and the coefficients led the method to results in the order of 10-11.

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Table 10 – Case 2: Impact of the coefficients on probability of failure

| Optimizer | Probability of failure – Case 1 | Probability of failure – Case 2 |
|---|---|---|
| SQP | $9.60 \times 10^{-4}$ | $1.28 \times 10^{-5}$ |
| Interior-point | $9.23 \times 10^{-5}$ | $2.52 \times 10^{-6}$ |
| Active-set | $6.35 \times 10^{-5}$ | $6.98 \times 10^{-11}$ |

Source: Elaborated by the authors, 2024

## 4 CONCLUSIONS

Based on the results obtained, the following conclusions can be drawn:

- The optimization methods showed good practical viability in the search for cost reduction in the simple bending problem of reinforced concrete beams, discussed herein.
- Incorporating the probability of failure leads to results that enable better analysis by the structural designer as it considers variability through the parameters.
- The SQP method showed the best results, from the point of view of both cost reduction and convergence.
- The active-set method resulted in higher costs and rather problematic convergence, requiring longer computational time without viability.
- Incorporating safety coefficients, as required by ABNT NBR 6118:2023, leads to considerable reductions in the probability of failure, ensuring better reliability.

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